basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2015

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 9x + 20 = 0$ (3)

1.1.2 $3x^2 + 5x = 4$ (correct to TWO decimal places) (4)

1.1.3 $2x^{\frac{-5}{3}} = 64$ (4)

1.1.4 $\sqrt{2-x} = x - 2$ (4)

1.1.5 $x^2 + 7x < 0$ (3)

1.2 Given: $(3x - y)^2 + (x - 5)^2 = 0$

Solve for x and y. (4)

1.3 For which value of k will the equation $x^2 + x = k$ have no real roots? (4)

[26]

QUESTION 2

The following geometric sequence is given: 10 ; 5 ; 2,5 ; 1,25 ; ...

2.1 Calculate the value of the 5^{th} term, T_5, of this sequence. (2)

2.2 Determine the n^{th} term, T_n, in terms of n. (2)

2.3 Explain why the infinite series 10 + 5 + 2,5 + 1,25 + ... converges. (2)

2.4 Determine $S_\infty - S_n$ in the form ab^n, where S_n is the sum of the first n terms of the sequence. (4)

[10]

QUESTION 3

Consider the series: $S_n = -3 + 5 + 13 + 21 + \ldots$ to n terms.

3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)

3.2 Write S_n in sigma notation. (2)

3.3 Show that $S_n = 4n^2 - 7n$. (3)

3.4 Another sequence is defined as:

$Q_1 = -6$
$Q_2 = -6 - 3$
$Q_3 = -6 - 3 + 5$
$Q_4 = -6 - 3 + 5 + 13$
$Q_5 = -6 - 3 + 5 + 13 + 21$

3.4.1 Write down a numerical expression for Q_6. (2)

3.4.2 Calculate the <u>value</u> of Q_{129}. (3)

[12]

QUESTION 4

Given: $f(x) = 2^{x+1} - 8$

4.1 Write down the equation of the asymptote of *f*. (1)

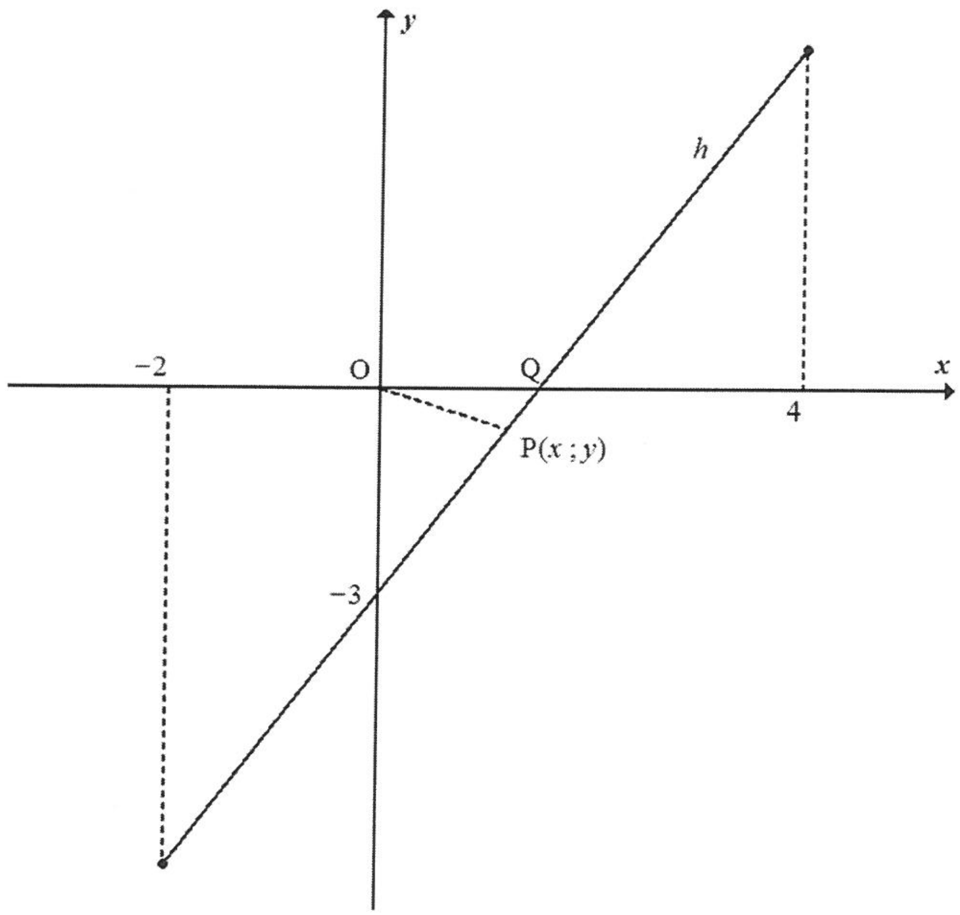
4.2 Sketch the graph of *f*. Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)

4.3 The graph of g is obtained by reflecting the graph of f in the y-axis. Write down the equation of g. (1)

[6]

QUESTION 5

Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The x-intercept of h is Q.



5.1 Determine the coordinates of Q. (2)

5.2 Write down the domain of h^{-1}. (3)

5.3 Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points. (3)

5.4 For which value(s) of x will $h(x) = h^{-1}(x)$? (3)

5.5 $P(x\ ;\ y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP. (5)

5.6 Given: $h(x) = f'(x)$ where f is a function defined for $-2 \leq x \leq 4$.

5.6.1 Explain why f has a local minimum. (2)

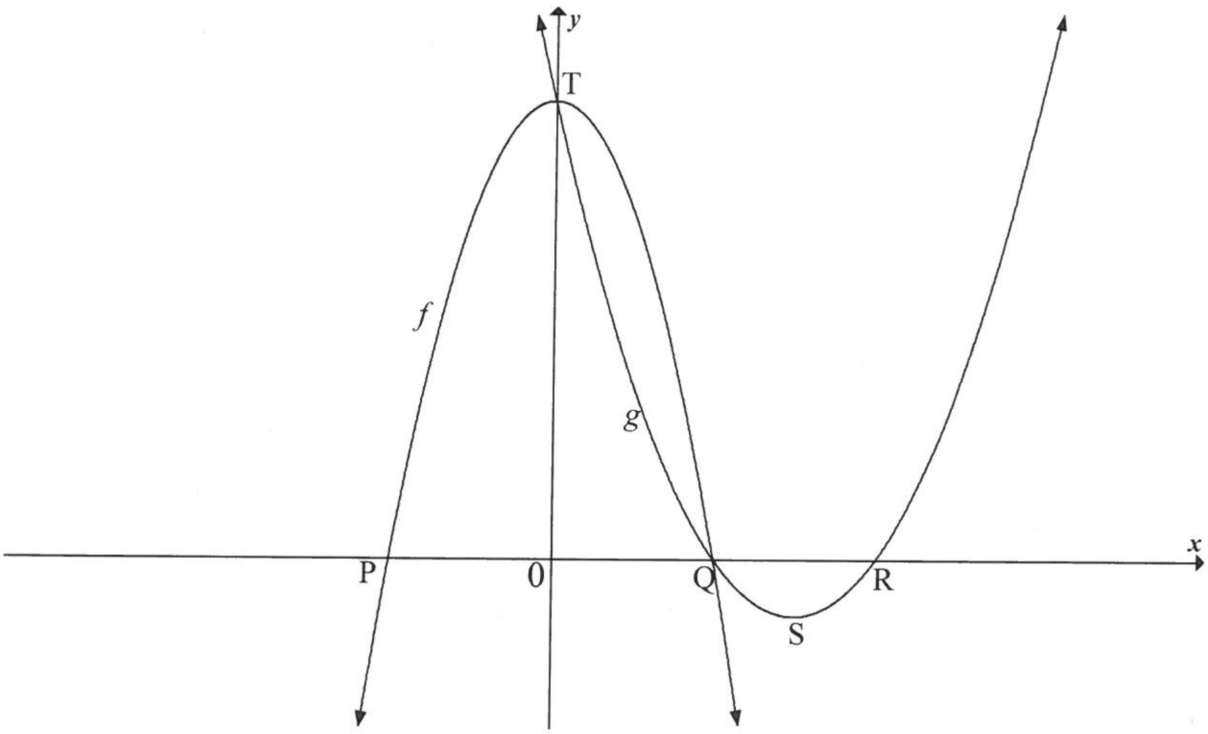
5.6.2 Write down the value of the maximum gradient of the tangent to the graph of f. (1)

[19]

QUESTION 6

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the x-intercepts of f. Points Q and R are the x-intercepts of g. S is the turning point of g. T is the y-intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of x for which $g''(x) > 0$. (2)

6.2 The function defined as $y = \dfrac{a}{x+p} + q$ has the following properties:

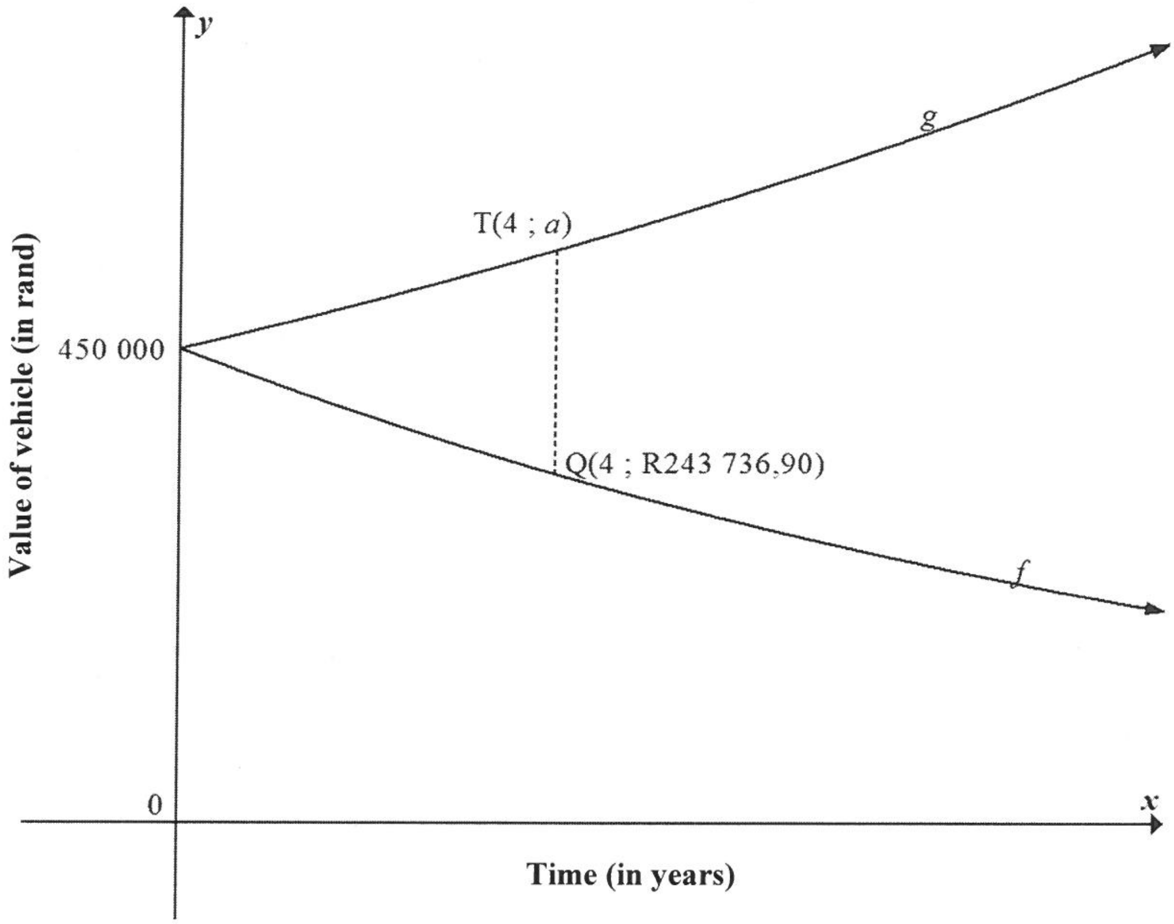
- The domain is $x \in R,\ x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R,\ x \neq -2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any. (4)

[12]

QUESTION 7

The graph of f shows the book value of a vehicle x years after the time Joe bought it.
The graph of g shows the cost price of a similar new vehicle x years later.



7.1 How much did Joe pay for the vehicle? (1)

7.2 Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought. (4)

7.3 If the average rate of the price increase of the vehicle is 8,1% p.a., calculate the value of a. (3)

7.4 A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13^{th} month and the last payment will be made at the end of the 48^{th} month. The sinking fund earns interest at a rate of 6,2% p.a., compounded monthly.

Calculate the monthly payment to the fund. (5)

[13]

QUESTION 8

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ (3)

8.2.2 $D_x\left(\frac{x^3 - 1}{x - 1}\right)$ (3)

[11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q$(2\ ;\ 10)$ are the turning points of h. The graph of h passes through the origin.

9.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

9.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

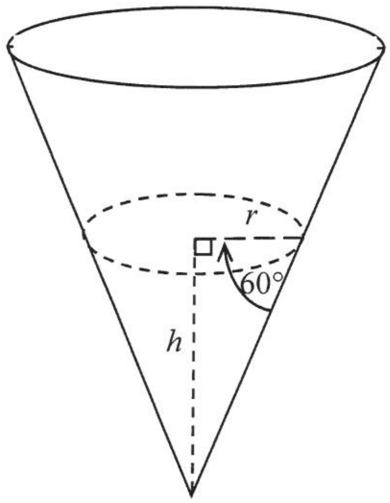
9.4 Explain the significance of the change in QUESTION 9.3 with respect to h. (1)

9.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)

[17]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60°, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\qquad$ $V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\qquad$ $V = \frac{4}{3}\pi r^3$

10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

[7]

QUESTION 11

11.1 For two events, A and B, it is given that:

$$P(A) = 0{,}2$$
$$P(B) = 0{,}63$$
$$P(A \text{ and } B) = 0{,}126$$

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

11.2 The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

11.2.1 Letters may be repeated (2)

11.2.2 Letters may not be repeated (2)

11.2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed (4)

11.3 There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6)
[17]

TOTAL: 150

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $A=P(1-ni)$ $A=P(1-i)^n$ $A=P(1+i)^n$

$T_n=a+(n-1)d$ $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$T_n=ar^{n-1}$ $S_n=\frac{a\left(r^n-1\right)}{r-1}$; $r\neq 1$ $S_\infty=\frac{a}{1-r}$; $-1<r<1$

$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $y-y_1=m(x-x_1)$ $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\tan\theta$

$$(x-a)^2+(y-b)^2=r^2$$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$$a^2=b^2+c^2-2bc.\cos A$$

$$area\ \Delta ABC=\frac{1}{2}ab.\sin C$$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$ $\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum fx}{n}$ $\sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$

$\hat{y}=a+bx$ $b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$